Perfect fluid flow
from
granular jet impact

Wendy W. Zhang

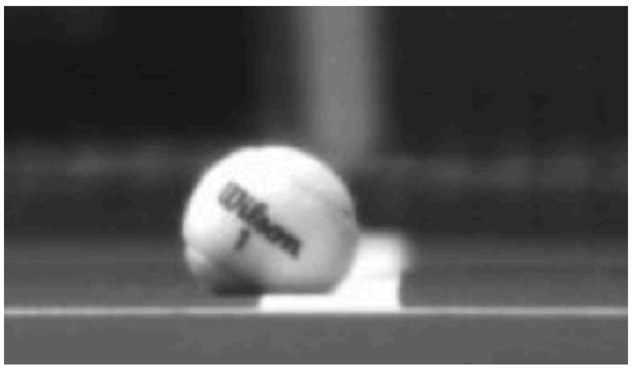
Physics Department & James Franck Institute
University of Chicago

Institute for Computing Science (ICis)
Verification, Validation and Uncertainty Quantification across Disciplines
Park City, Utah August 2011

Overview

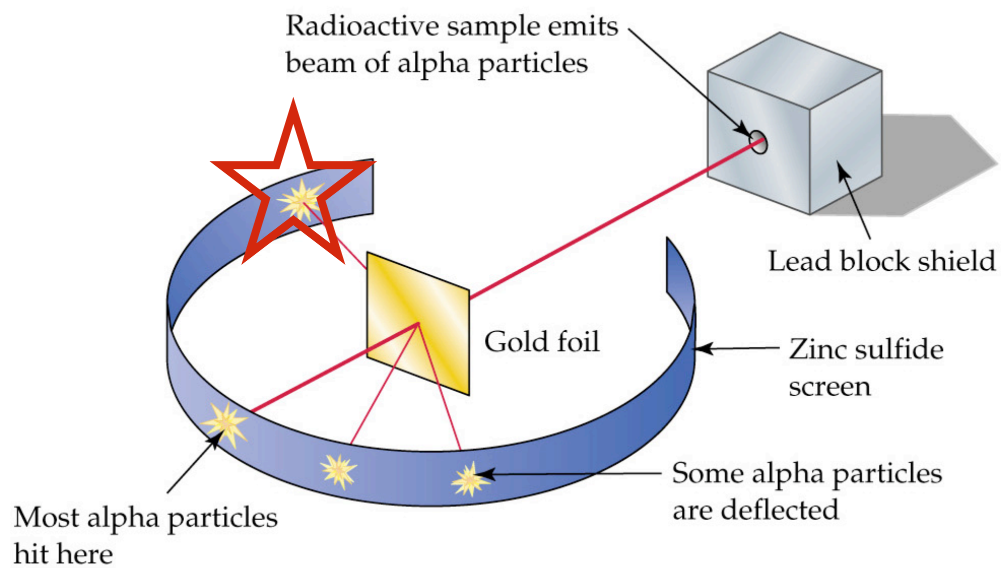
Impact is familiar, important and a powerful experimental tool

mechanics
tennis ball hitting court line



Hawkeye innovations

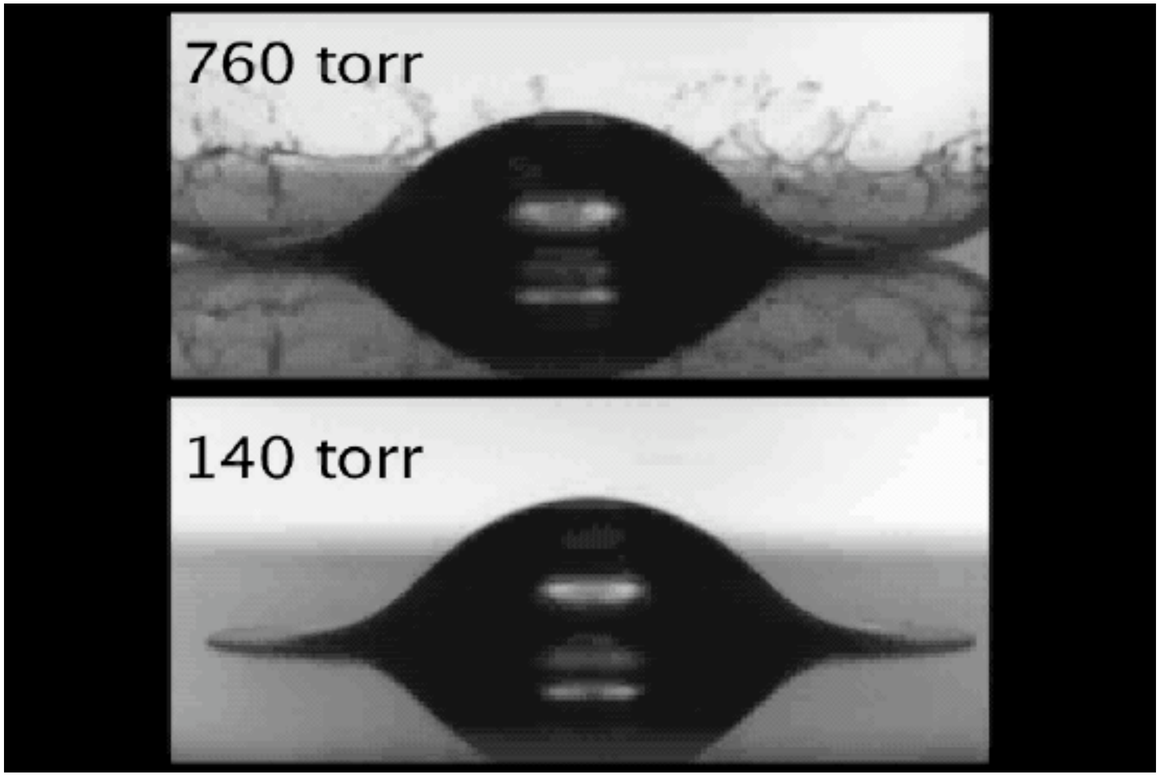
high-energy experimental physics
impact ➔ scattering ➔ structure



Rutherford's goldfoil scattering experiment
wikipedia

Overview

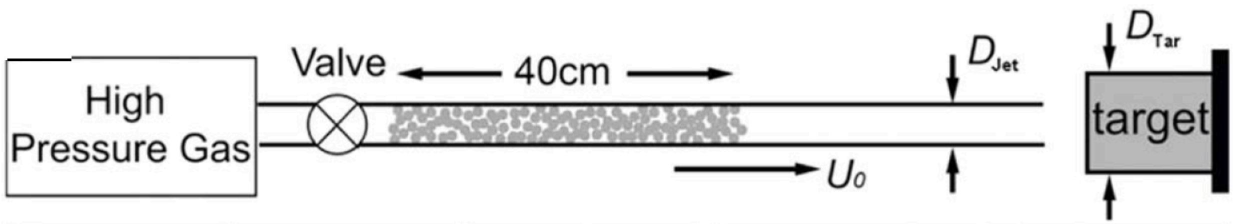
Impact can be surprising



4 mm diameter ethanol drop, impact speed = 4 m/s

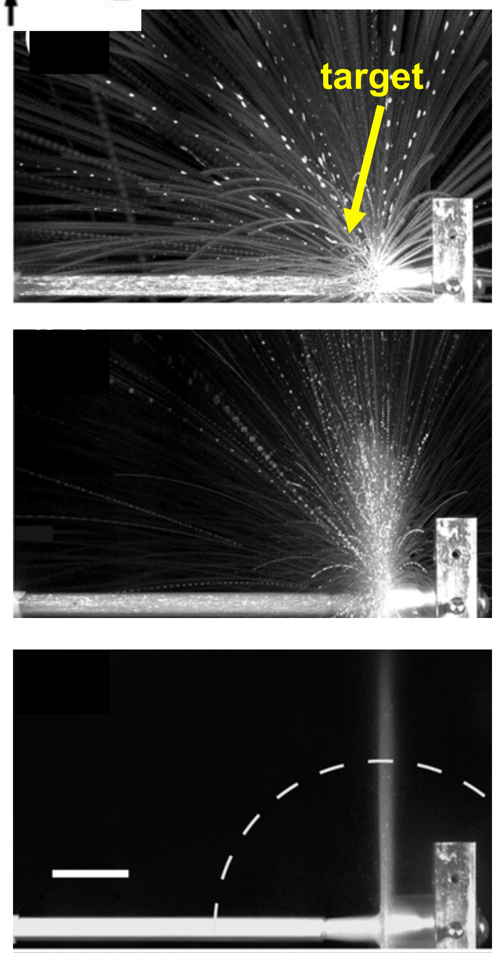
How does air create a splash?

Granular jet impact
emergence of liquid-like behavior



Cheng et al. PRL 07

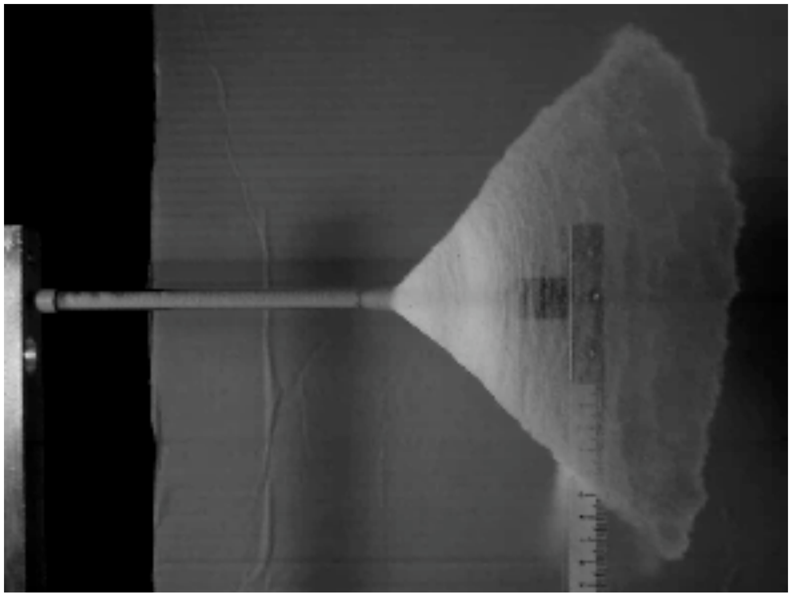
loosely packed jet
➔ shower of recoils



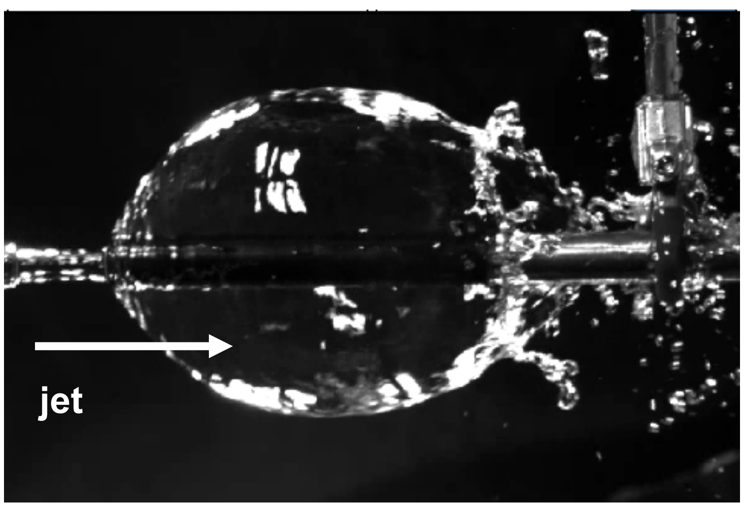
target holder

dense jet
➔ejecta
collimated
into thin sheet

Impact of thick jet onto small target



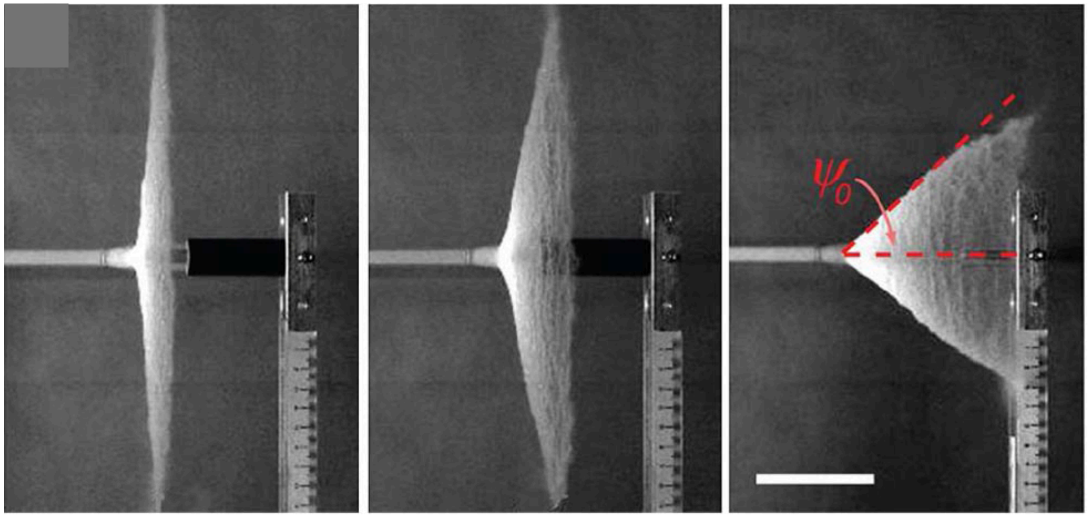
➔ *hollow conical sheet*

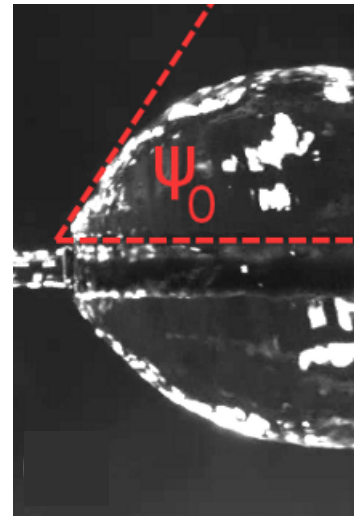


Water jet impact
➔ *hollow water bell*

Granular "water bell"?

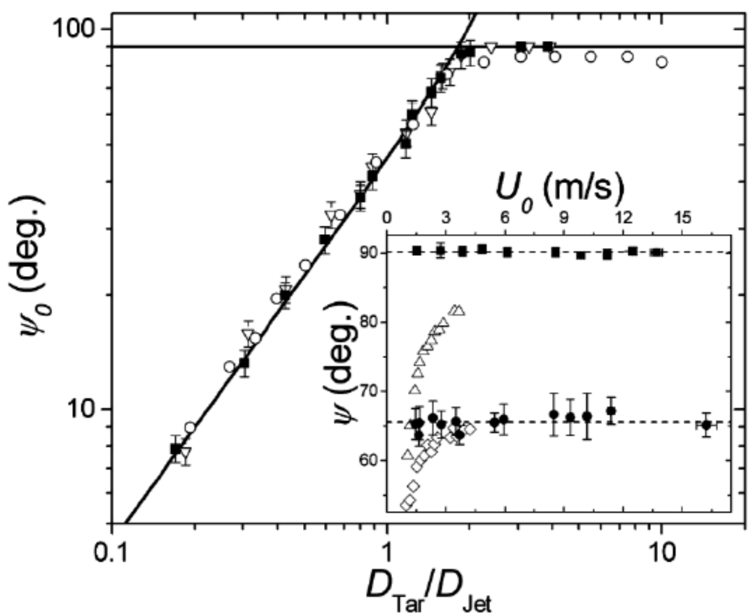
Ejecta sheet angle changes with D_{Target}/D_{Jet}





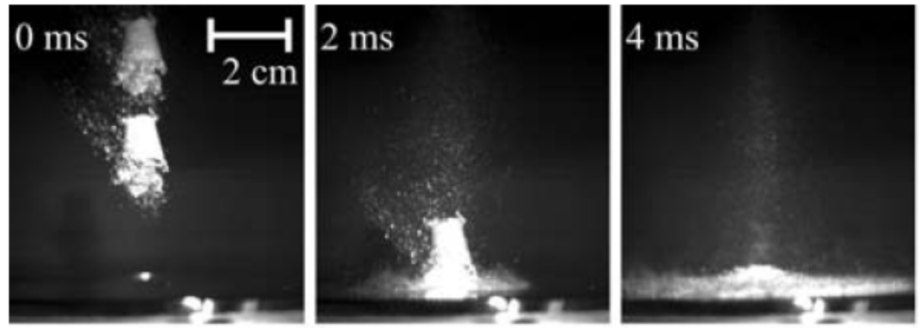
reducing D_{Target}/D_{Jet}

Granular ejecta angle ψ_0 agree numerically with values for water jet ➔ liquid-like behavior

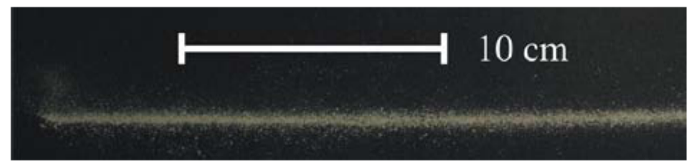


Context: emergence of collimated ejecta

• Formation of planetismals via collison of dust aggregates



ejecta collimated within 1˚



• Collimated ejecta from collision of gold ion jets at relativistic speeds

-- Have been interpreted as evidence for a liquid quark-gluon phase

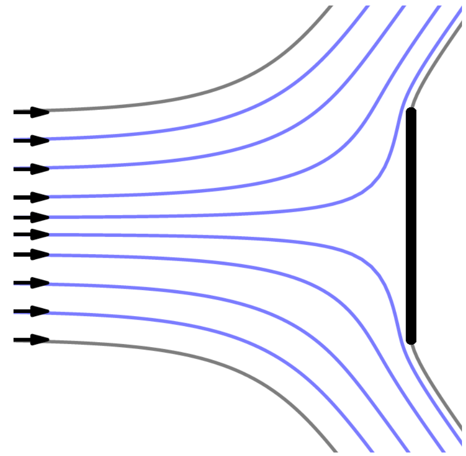
Teiser & Wurm, Mon. Not. R. Astron. Soc. 2009
Pozkanser, Voloshin, Ritter... 2008 APS Bonner prize talk
Romatschke & Romatschke PRL 2007

Interior motion
different from water jet impact

impact of water jet

continous motion

no dead zone

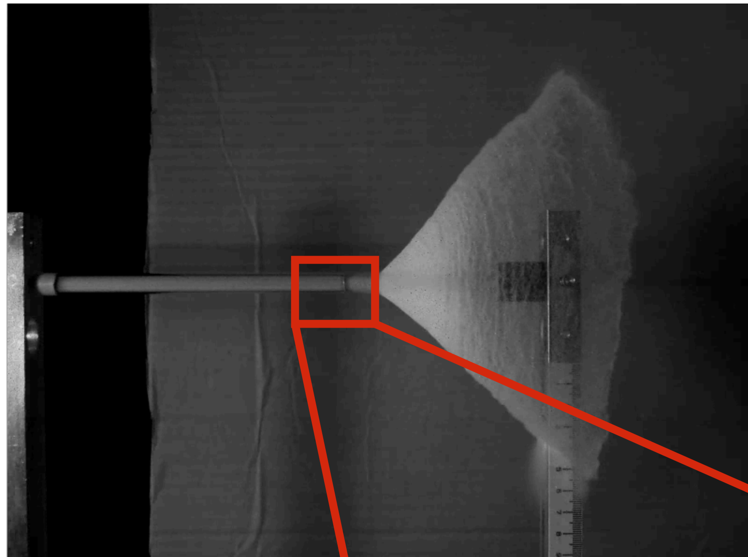


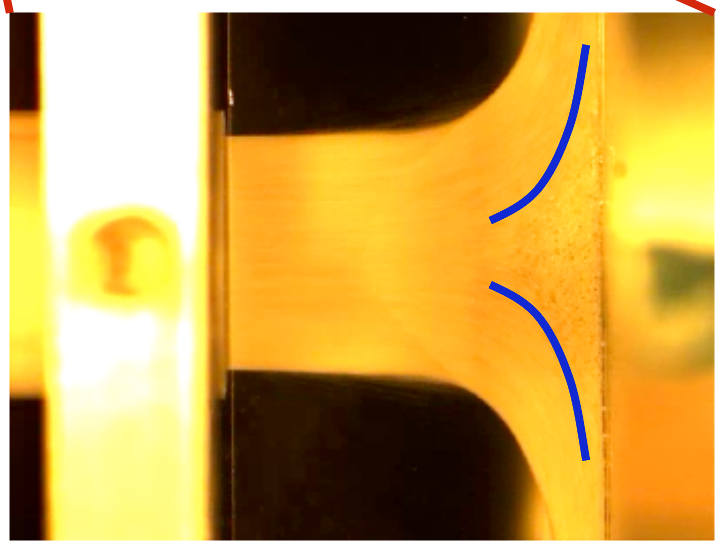
impact of granular jet

flowing & static region

dead zone

Liquid-like ejecta co-exists with solid-like interior





Ejecta agree quantitatively despite dissimilar interior

How?

Maybe
high-density granular impact generically produces liquid-like ejecta, regardless of nature of interior state

Suppose we get rid of dead zone, would we still see collimated ejecta

Need numerics

the plan
1. Reproduce dead zone & ejecta sheet
2. Varying parameters to get rid of dead zone

Simulation

jet →

QuickTime™ and a
decompressor
are needed to see this picture.

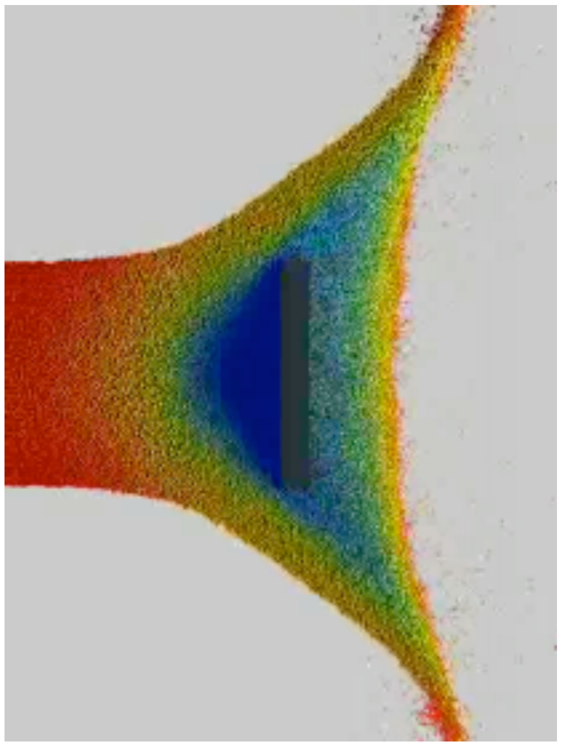
red = high speed
blue = zero speed

Minimal Physics Model

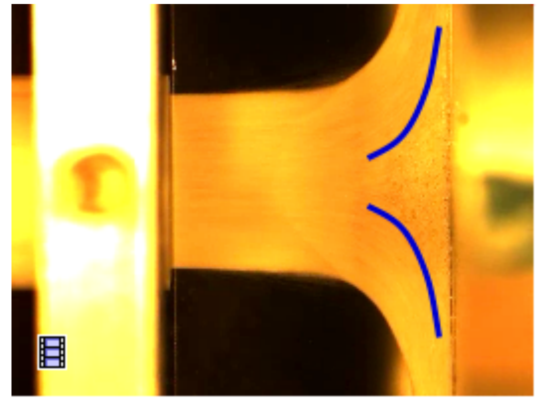
- perfectly rigid grains
- spheres
- inelastic collisions
- friction between grains
- friction at target

Simulation vs. experiment

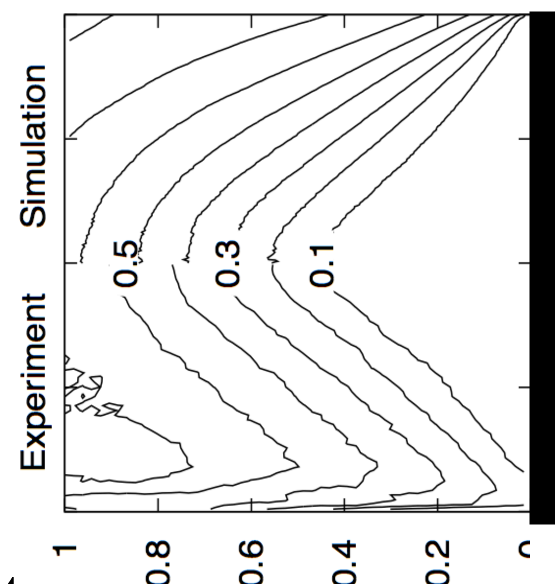
jet ⟶



red = high speed
blue = zero speed

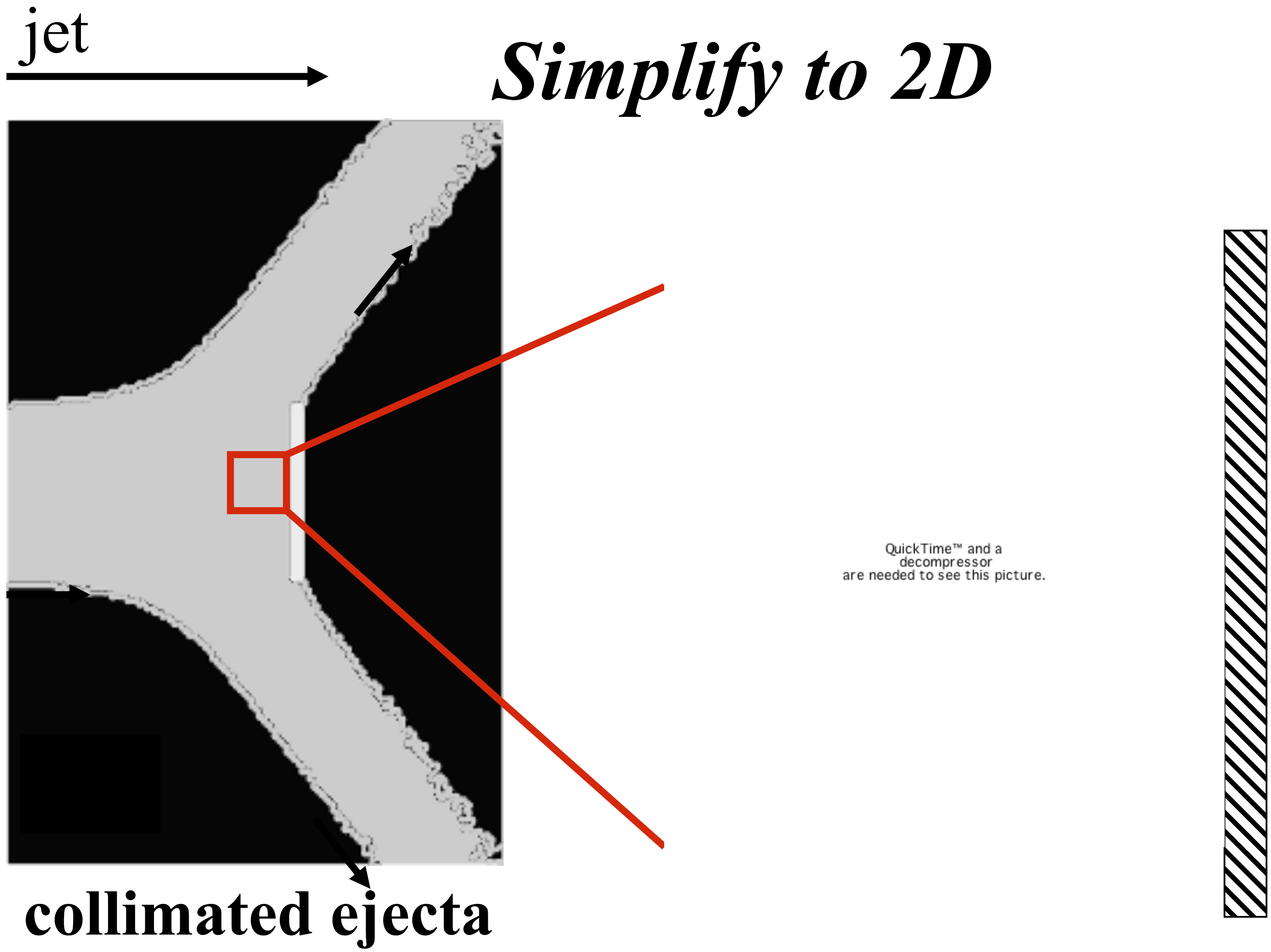


velocity contour comparison



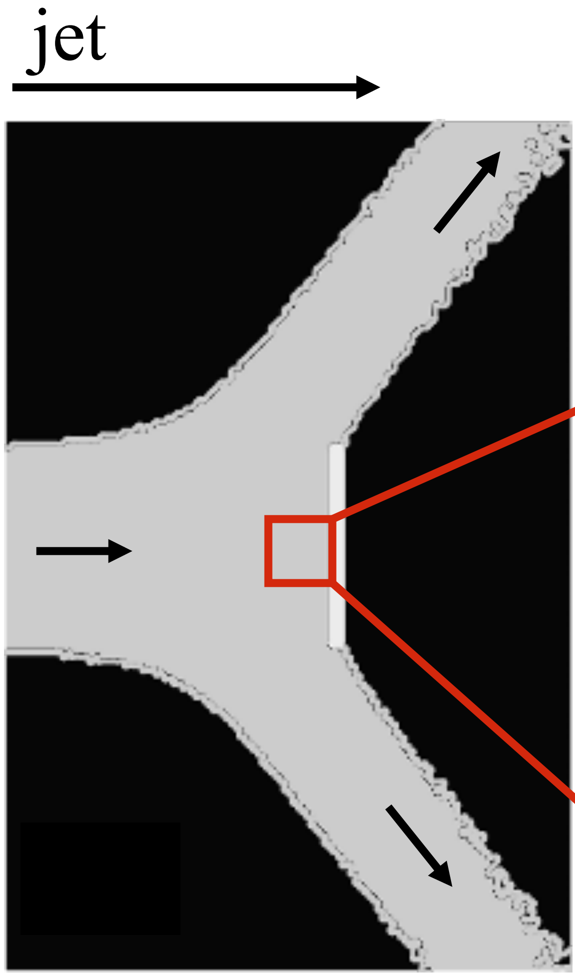
Quantitative agreement

Simplify to 2D



Same impact dynamics

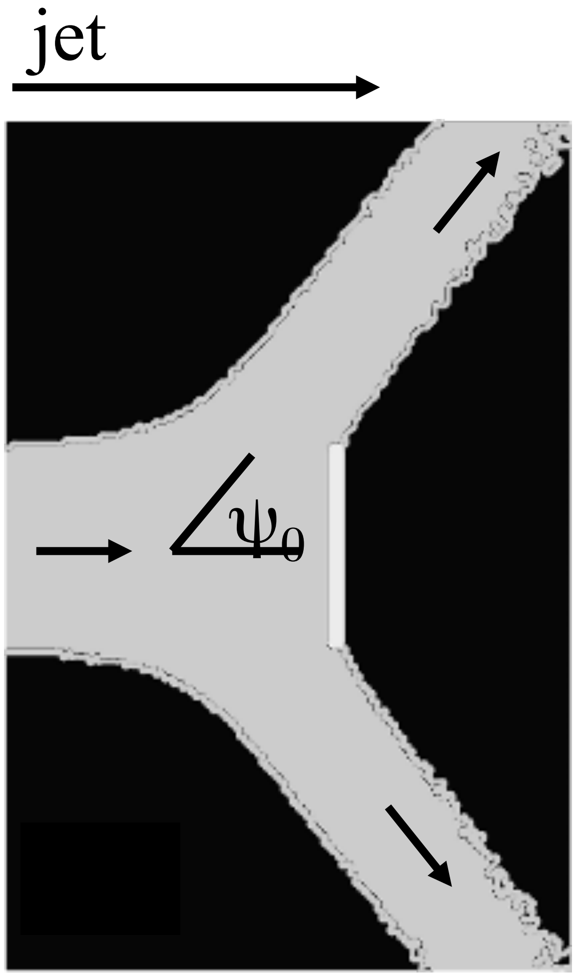
Frictionless target (2D)



collimated ejecta

liquid-like response without dead zone

Collimated ejecta is generic



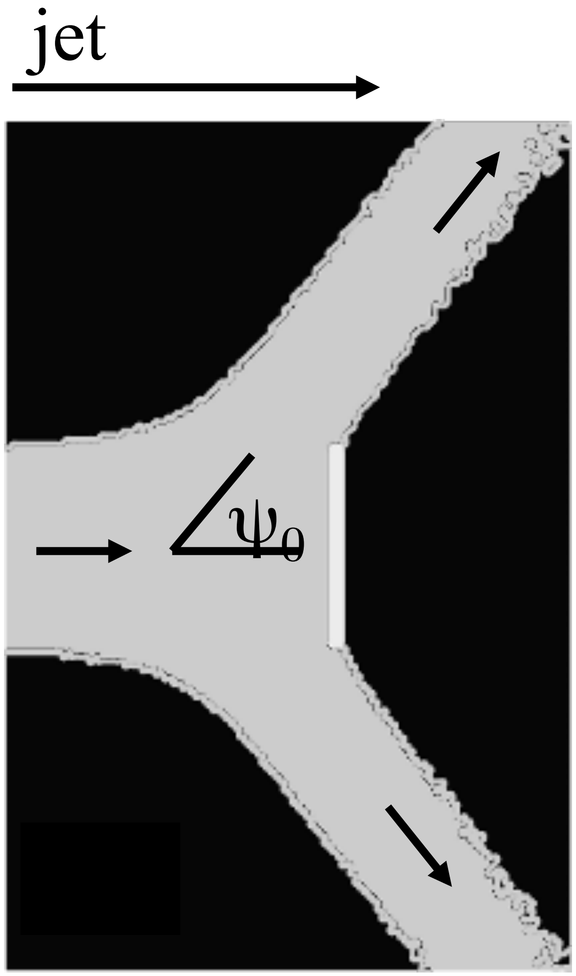
2D ejecta angle changes slightly

52° (with dead zone)

➔ 47° (without deadzone)

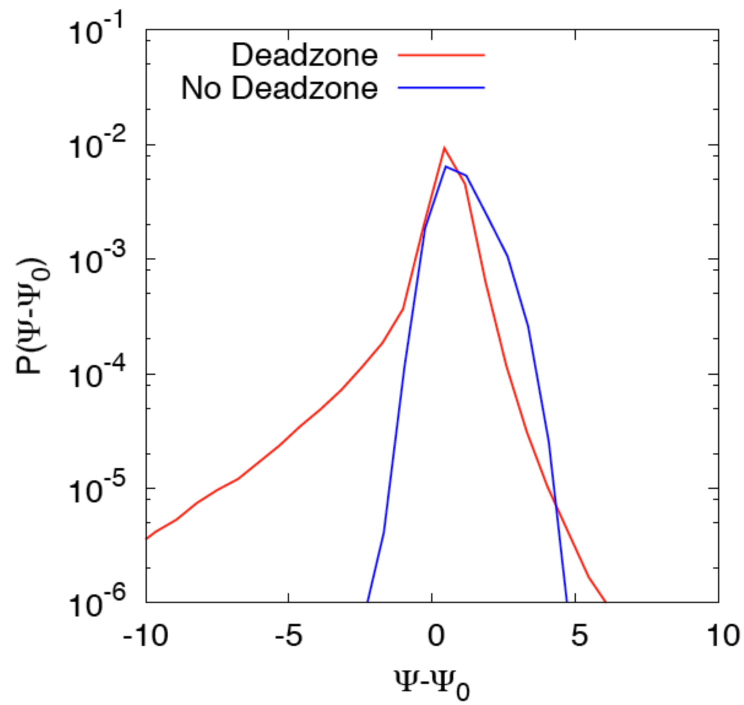
3D also slight change

Collimated ejecta is generic

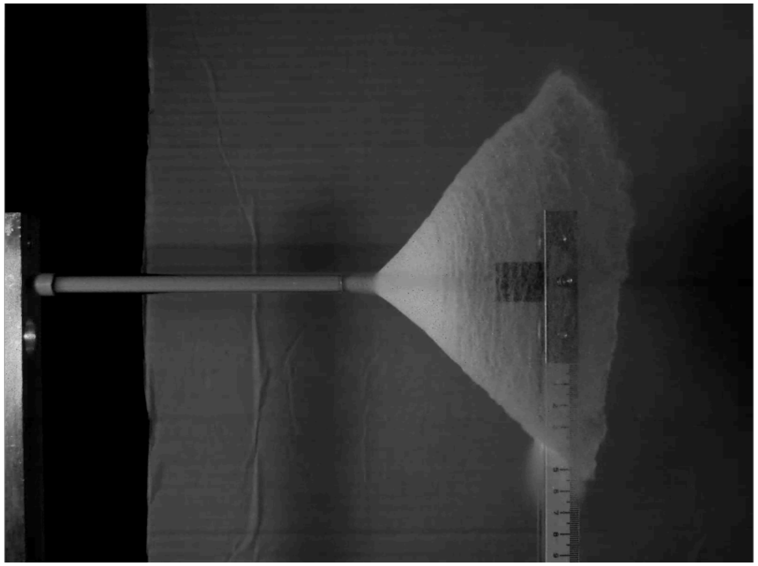


3D simulation

probability of grain ejected at angle $\psi - \psi_0$



Why liquid-like ejecta?



Granular & water jet impact controlled by same idealized limit of perfect fluid flow

Perfect fluid flow contains no information

Demonstrating connection between granular impact and perfect fluid flow

granular impact

inelastic / friction

no cohesion

2D simulation

detailed quantitative comparison

perfect fluid impact

no dissipation

no surface tension

2D exact solution

How good should the agreement be?

granular impact

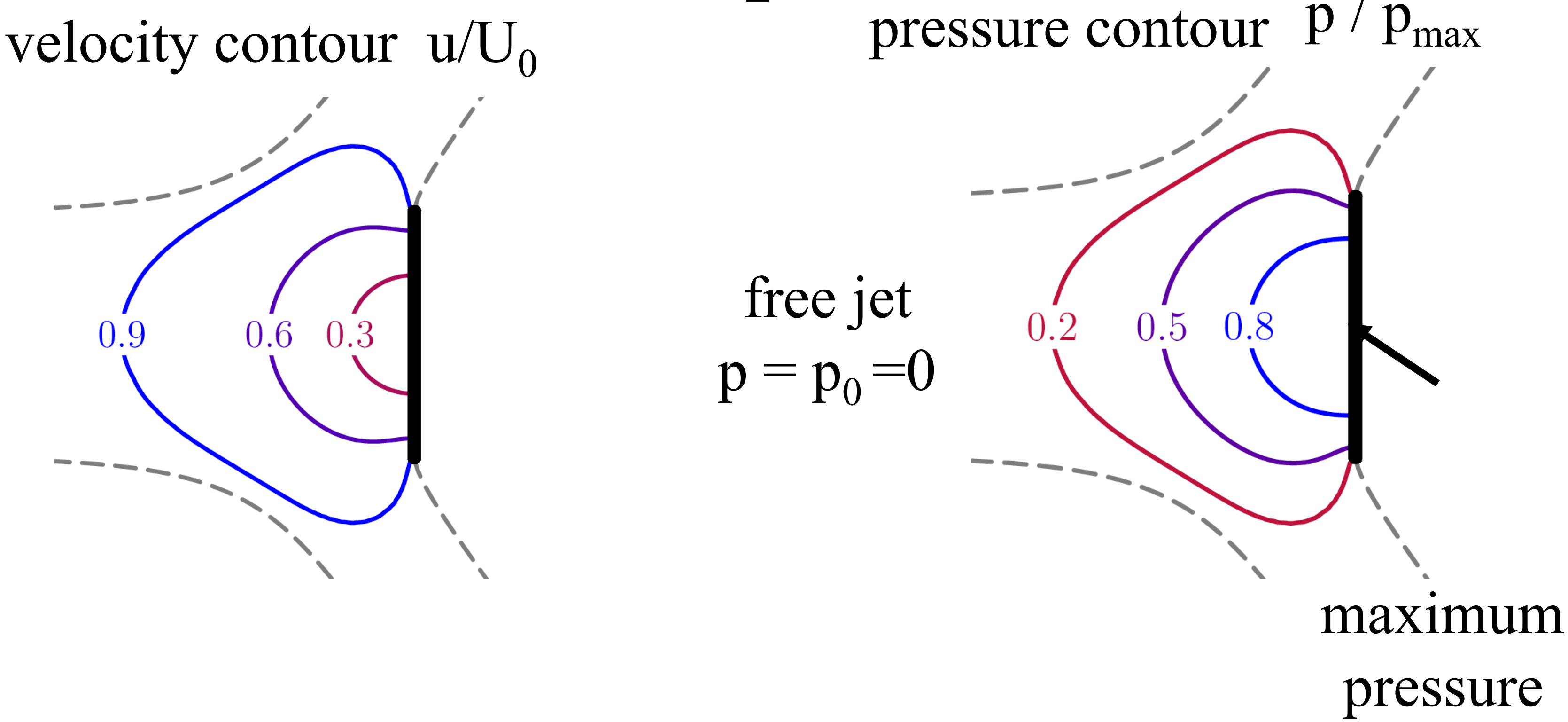
inelastic / friction

perfect fluid impact

no dissipation

expect granular ejecta to move slower

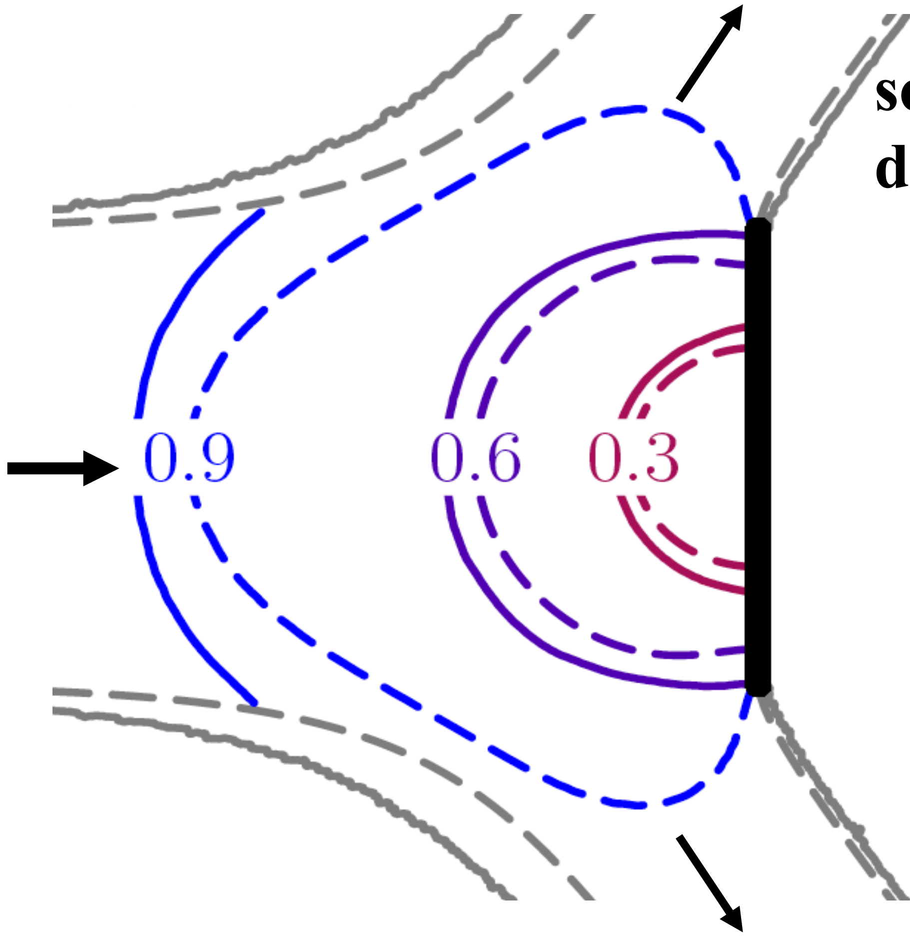
Exact solution for perfect fluid impact



Velocity contours

local speed / impact speed

solid line = granular simulation
dashed line = perfect fluid solution

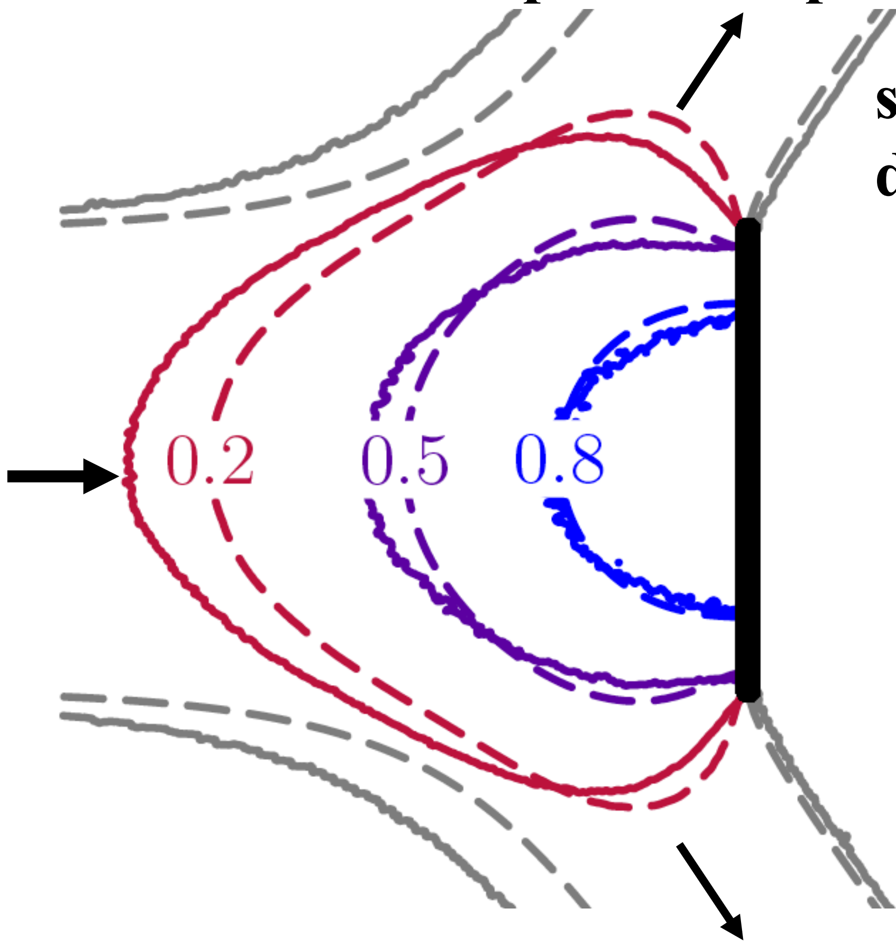


Inelasticity & friction
➔ejecta wider than perfect fluid

Pressure contours

local pressure / pressure at target center

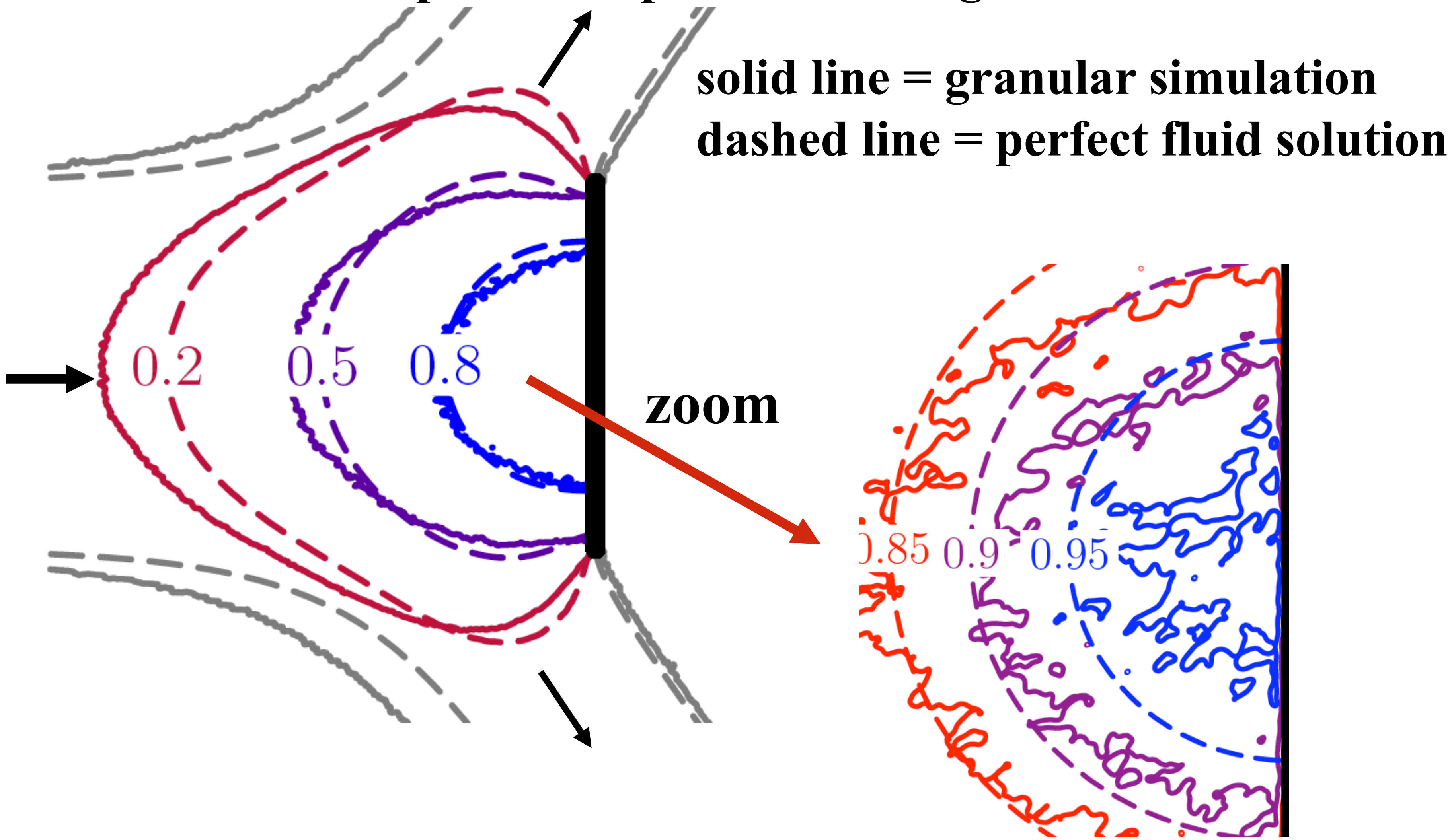
solid line = granular simulation
dashed line = perfect fluid solution



Quantitative agreement

Pressure contours

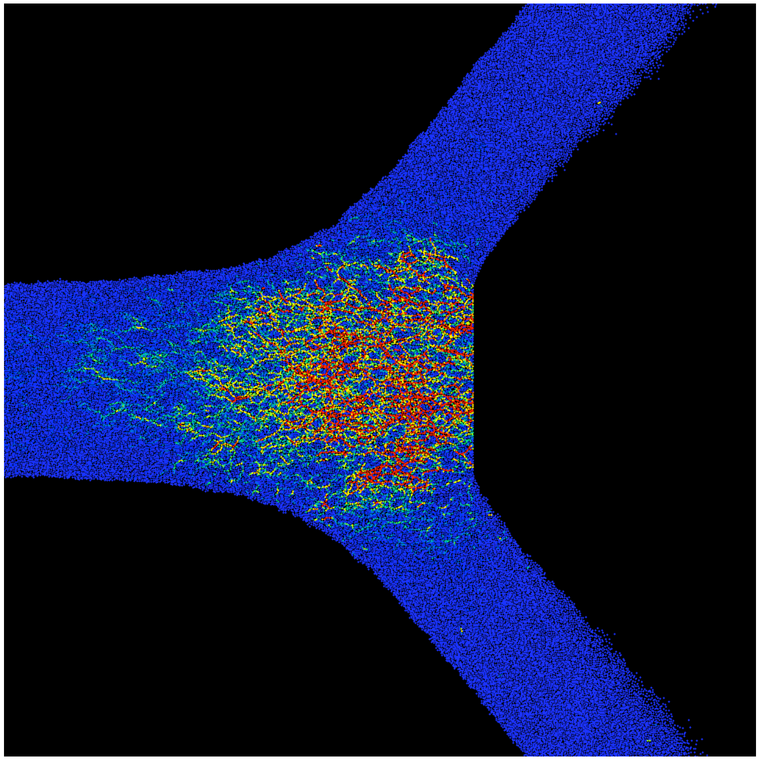
local pressure / pressure at target center



Larger fluctuations

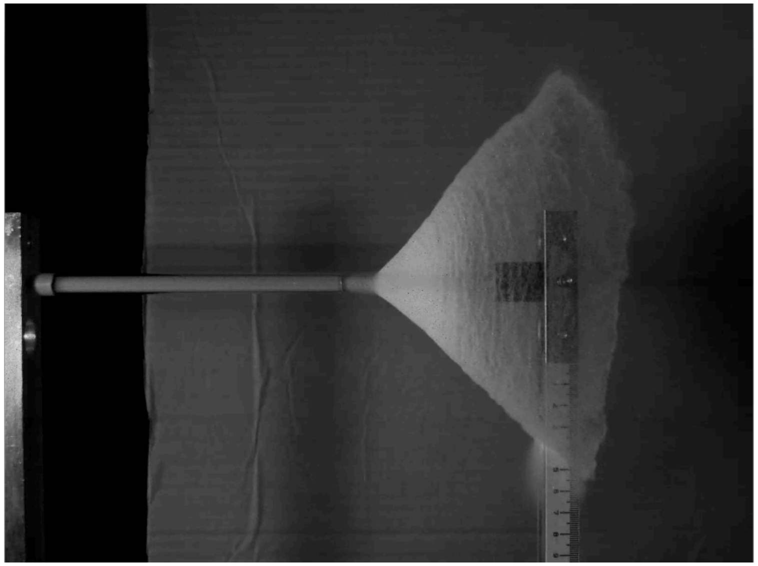
Distribution of compressive forces on grains

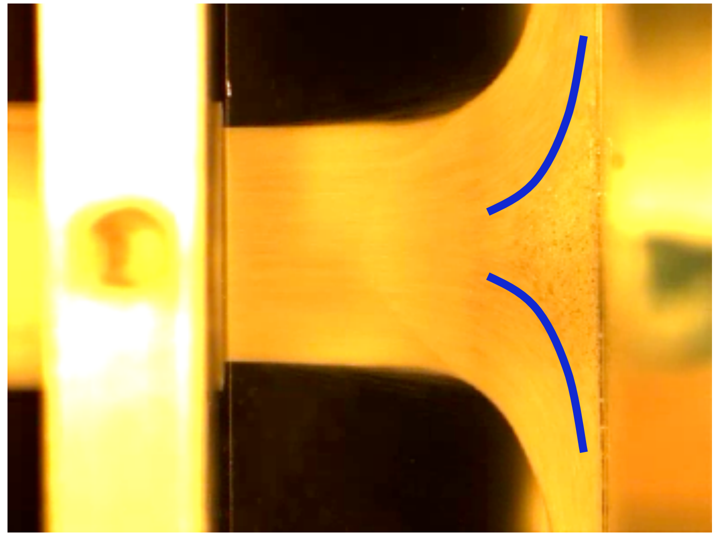
red = large blue = zero



force chains ➔ fluctuations about perfect fluid behavior

Why liquid-like ejecta?





Generic outcome of high density impact

➔ Collimated, liquid-like ejecta

With Nicholas Guttenberg, Jake Ellowitz,

Herve Turlier, Sidney R. Nagel

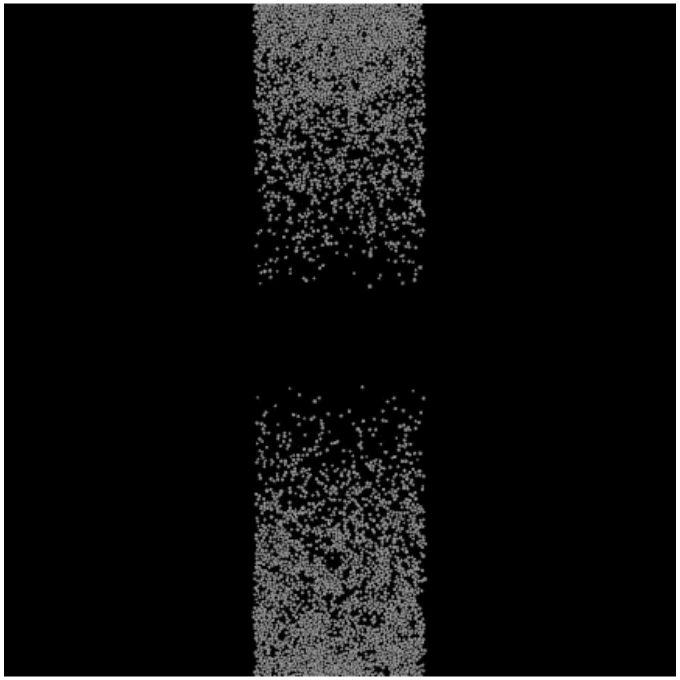
Acknowledgements: Xiang Cheng, Efi Efrati, Heinrich M. Jaeger

NSF-MRSEC, Keck Foundation, NSF-CBET

Thank you

[redacted]

What next?



Simulating dense granular impact is hard

standard toolkit

kinetic regime (gas-like)

event-driven, hard-particle dynamics
instantaneous collision
exact momentum & energy conservation
✖ vulnerable to numerical singularity (inelastic collapse)

quasi-static, dense packing

discrete-element method
elastic energy penalizing overlap
✖ particles much softer than real materials
✖ does not conserve energy exactly

Simulating dense granular impact is hard

standard toolkit

kinetic regime (gas-like)

event-driven, hard-particle dynamics
instantaneous collision
exact momentum & energy conservation
✖ vulnerable to numerical singularity
(inelastic collapse)

dense flow conserving momentum & energy

quasi-static, dense packing

discrete-element method
elastic energy penalizing overlap
✖ particles much softer than real materials
✖ does not conserve energy exactly

How we simulate dense granular impact

kinetic regime (gas-like)

event-driven, hard-particle dynamics
instantaneous collision
exact momentum & energy conservation
✖ vulnerable to numerical singularity
(inelastic collapse)

Modified event-driven dynamics

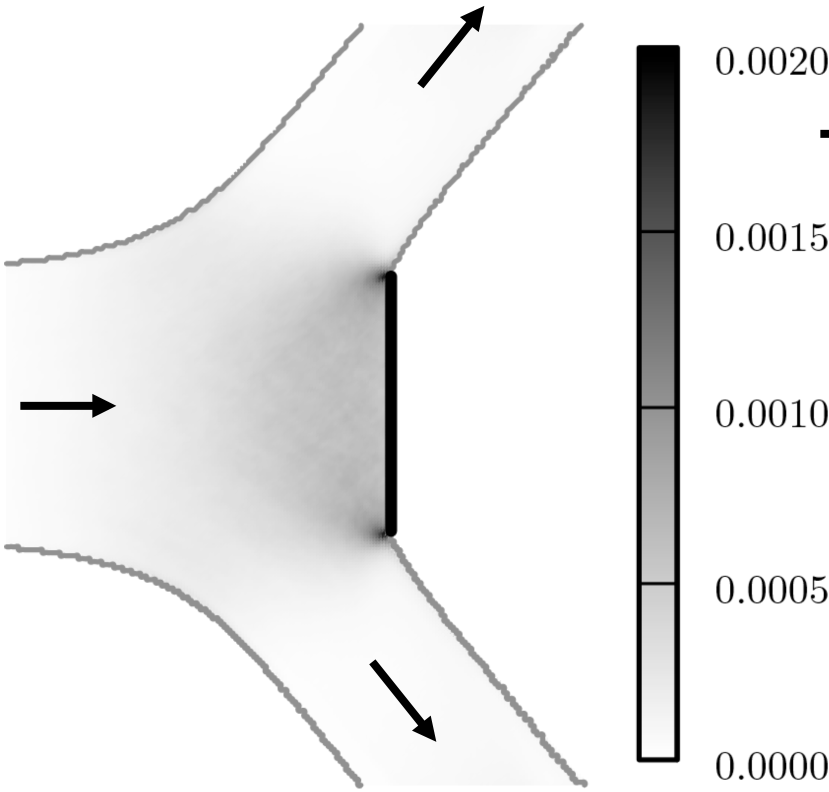
Evolve dynamics in fixed time interval Δt
At time t find all particles that will overlap during Δt
Avoid overlap by pretending the particles have collided at t
Evolve collisions

Iterate until no overlap occurs in Δt
Evolve to next time-step

McNamara, Flekk y & M al y
PRE 2000

Guttenberg arXiv:1102.2483v1

Energy budget



$$\frac{\textbf{fluctuation in kinetic energy density}}{\textbf{total energy density}}$$

Dominated by mean flow

Mass & momentum budget

Nearly incompressible flow

velocity field

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \Rightarrow \underline{\nabla \cdot \mathbf{u} = 0}$$

density

constant & uniform density

High impact speed, large deceleration

→neglect dissipation

→ momentum transport has only inertia

pressure field

$$\underline{\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} + \nabla p = 0}$$

Perfect fluid flow

Only pressure gradient

No shear stress

Pressure in granular impact

Define time interval τ << impact time-scale
Define sample region (~ 5 particles wide)
Sum impulses I_n experienced by each particle
Define stress component as

$$\sigma_{ij} \propto \frac{1}{\tau} \sum_n (\mathbf{I}_n \cdot \hat{\mathbf{e}}_i)(\hat{\mathbf{r}}_n \cdot \hat{\mathbf{e}}_j), \quad i, j \in \{x, y\}$$

center-of-mass vector between colliding particles

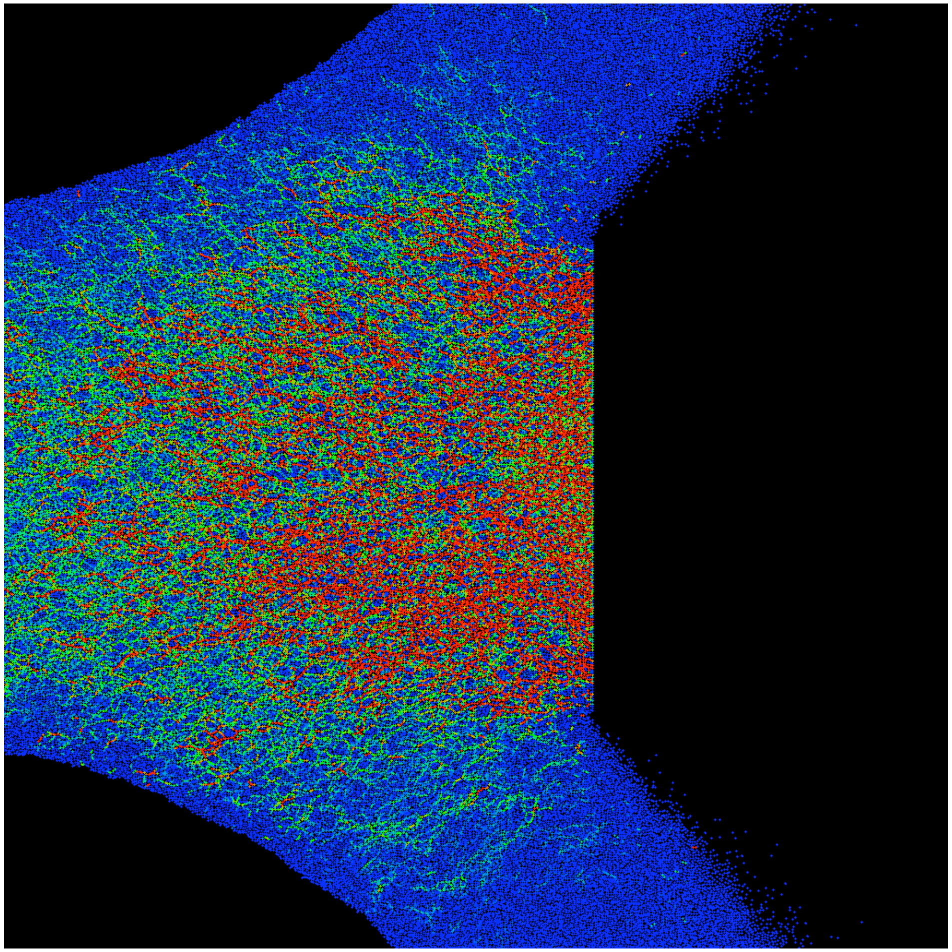
Define pressure as

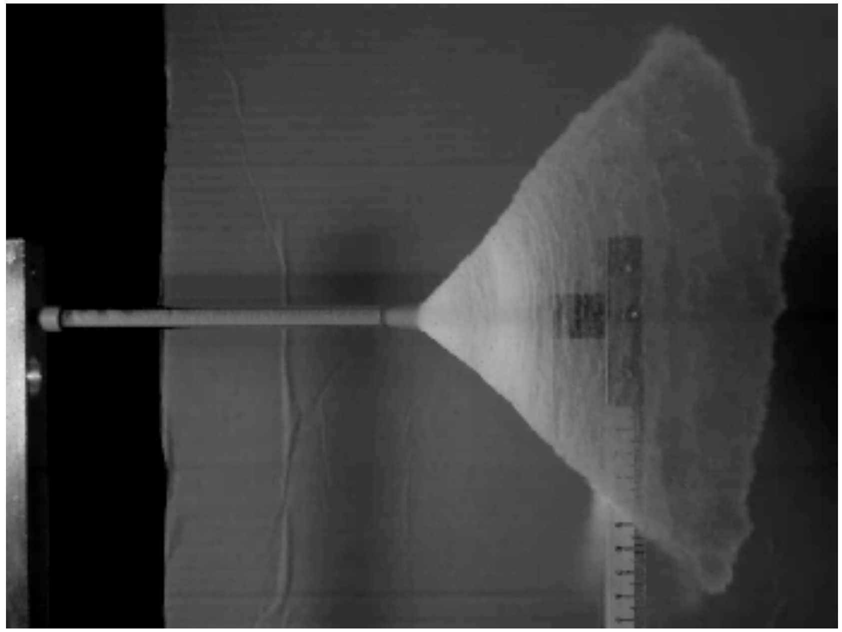
$$P = (|\sigma_{xx}| + |\sigma_{yy}|)/2$$

Average over T >> impact time-scale for contours

Distribution of compressive forces on grains with dead zone present

red = large   blue = zero





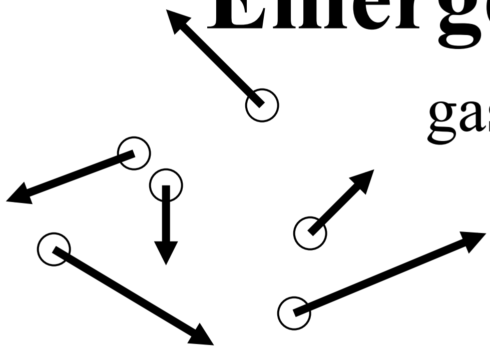
Granular jet impact

➔ strongly coupled liquid?

When does liquid-like behavior emerge?

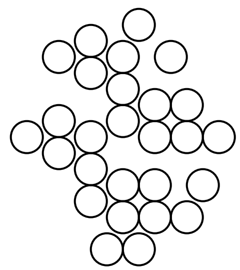
Emergence of liquid-like behavior
child's view

gas



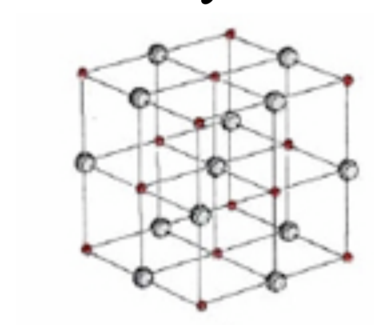
• fills available volume
• flows under shear

liquid



• fixed volume
• flows under shear

crystalline solid

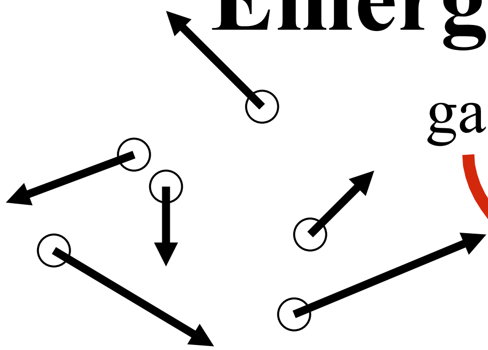


• fixed volume
• resists shear

an intermediate state of matter

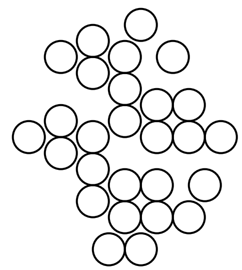
Emergence of liquid-like behavior
traditional view

gas



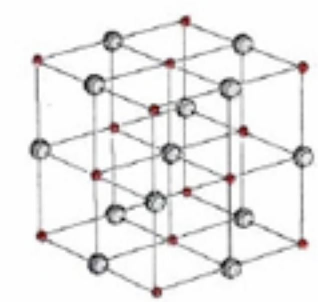
• fills available volume
• flows under shear

liquid



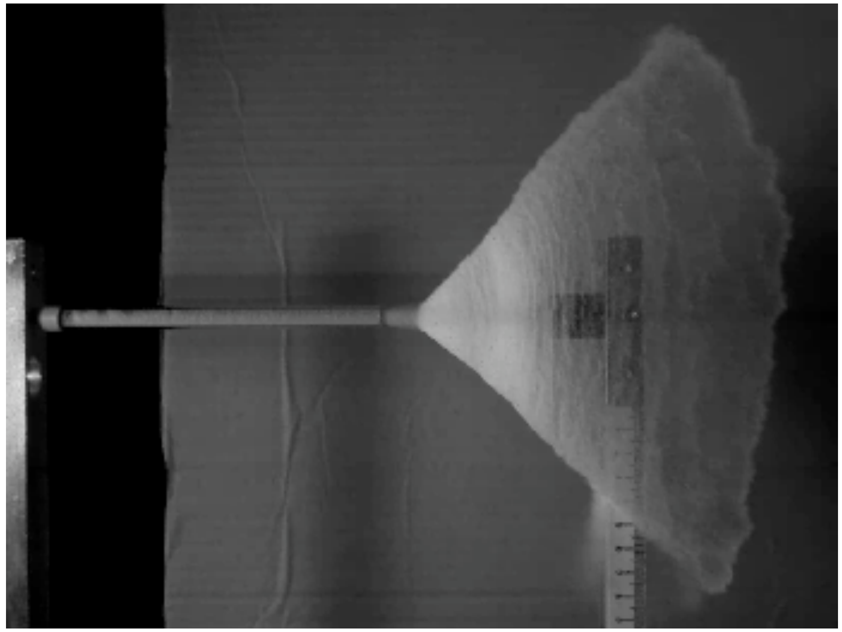
• fixed volume
• flows under shear

crystalline solid



• fixed volume
• resists shear

attractive interaction ➔ liquid



Cohesion between particles effectively zero

Emergence of liquid-like behavior
modern view

In a liquid
spatial arrangement of molecules is controlled

- mostly by strong repulsion between nearby neighbors
  - ➔ can model molecules as hard spheres
- attraction perturbs microscopic spatial structure
  - ➔ can model attraction by confinement

➔ computation scheme yielding structure & equation of state

confine hard spheres ➔ liquid

Velocity along centerline